

MATHEMATICS

Class: X

REAL NUMBERS

Answer the following:

1. Show that the square of any positive integer is of the form $3m$, or $3m + 1$ for some integer m .
2. Use Euclid's lemma to division show that the cube of any positive integer is either of the form $9m$, $9m + 1$, $9m + 8$ for some integer m
3. Show that the cube of any positive integer is of the form $4m$, $4m + 1$ or $4m + 3$ for some integer m
4. Show that any positive odd integer is of the form $4m + 1$ or $4m + 3$ where m is some integer.
5. Prove that the product of any three consecutive positive integers is divisible by 6
6. Express the integers as the product of its prime factors (i) 13915 (ii) 556920
7. By using Euclid's algorithm, find the largest number which divides 650 and 1170.
8. Use Euclid's division algorithm to find the HCF of 4052 and 420
9. Find the HCF of 255 and 867 by Euclid's division algorithm.
10. Find the HCF of 865 and 255 using Euclid's division lemma.
11. Find the largest number that divides 2053 and 967 and leaves a remainder of 5 and 7 respectively.
12. Find the HCF and LCM of 26 and 91 and verify that $\text{LCM} \times \text{HCF} = \text{Product of the integers}$.
13. During a sale, colour pencils were being sold in packs of 24 each and crayons in packs of 32 each. If you want full packs of both and the same number of pencils and crayons, how many of each would you need to buy?
14. A mason has to fit a bathroom with square marble tiles of the largest possible size. The size of the bathroom is 10 ft by 8 ft. What would be the size in inches of the tile required that has to be cut and how many such tiles are required.
15. Three bells toll at intervals of 12 minutes, 15 minutes and 18 minutes respectively. If they start tolling together, after what time will they next toll together?
16. Two tankers contain 850 litres and 680 litres of petrol respectively. Find the maximum capacity of a container which can measure the petrol of either tanker in exact number of times.

17. An army contingent of a1000 members is to march behind an army band of 56 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?
18. A circular field has a circumference of 360 km. Three cyclists start together and can cycle 48, 60 and 72 km a day, round the field. When will they meet again?
19. If HCF of 144 and 180 is expressed in the form $13m - 3$, find the value of m .
20. Find the HCF of 65 and 117 and find a pair of integral values of m and n such that $\text{HCF} = 65m + 117n$.
21. (i) Check whether 6^n can end with the digit 0 for any natural number n .
(ii) Show that 12^n cannot end with digits 0 or 5 for any natural number 'n'.
(iii) Show that 9^n cannot end with digit 0 for any natural number n .
22. Determine the values of p and q so that the prime factorization of 2520 is expressible as $2^3 \times 3^p \times q \times 7$.
23. (i) Explain why $7 \times 11 \times 13 + 13$ is a composite number
(ii) Explain why $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ is a composite number
(iii) Explain whether the number $3 \times 5 \times 13 \times 46 + 23$ is a prime number or a composite number.
24. Find HCF of 378, 180 and 420 by prime factorization method. Is $\text{HCF} \times \text{LCM}$ of three numbers equal to the product of the three numbers?
25. Prove that $\sqrt{5}$ is an irrational number.
26. Prove that $\sqrt{2} + \sqrt{5}$ is irrational.
27. Prove that $5 - 3\sqrt{2}$ is an irrational number.
28. Show that $\frac{1}{\sqrt{2}}$ is an irrational number.
29. Show that the reciprocal $3 + 2\sqrt{2}$ is an irrational number.
30. Show that $(\sqrt{3} + \sqrt{5})^2$ is an irrational number.

